ISI – Bangalore Center – B Math - Physics II – Mid Semestral Exam

Date: 8th September 2016. Duration of Exam: 3 hours

Total marks: 40 ANSWER ALL QUESTIONS

**Q 1. [Total Marks: 4+5+2]**

a. ) An certain amount of gas undergoes adiabatic free expansion from $(P_i, V_i)$ to $(P_f, V_f)$. Prove using only the first law of thermodynamics that there is no quasistatic adiabatic process that can take the system from $(P_f, V_f)$ to $(P_i, V_i)$. Do not assume that it is an ideal gas.

b.) A large volume of ideal gas of total volume $V_i$ is kept in a tank constant pressure, higher than the atmosphere pressure $P$, in a tank as shown Fig 1. It is kept at the atmospheric temperature *T*. The gas tank is connected to a much smaller vacuum chamber of volume $V$. The vacuum chamber is adiabatically shielded from the large volume of gas as well as from the atmosphere. The stopcock is opened and gas quickly flows from the large tank into the vacuum chamber to equalize pressure. The stock cock is adiabatically closed so that there is no transfer of heat between the tank and the chamber. In the process, the mass *m* kept to maintain constant pressure drops by a height *h*. Let $v_i$ be the molar volume in the initial gas tank and $v$ be the molar volume in the vacuum chamber after it is filled with gas. Assume that the molar specific heats $c_v$, $c_p$ are constants.

Calculate the difference in temperature between the gas in the large tank and in the filled vacuum chamber in terms of *P, T, R, m, g, h,* $c_v$, $c_p$, $V_i$, $V_t$, $v_i$ and $v$? [Please note that the result may not depend on some of these parameters].

c.) Apply the result of part b.) for the case *m=0,* and show that the temperature in the vacuum chamber ( after it is filled by opening it to the atmosphere ) is given by $\frac{(c_v + R)}{c_v} T$.

**Q2. [Total Marks: 2+4]**

a. ) Show that a reversible engine that operates between two reservoirs at temp $T_1$ and $T_2$, cannot absorb heat from both reservoirs and convert it to work.

b.) Consider all engines that work using two reservoirs at temperatures $T_1$ and $T_2$ ( $T_2 > T_1$ )in which $Q_2$ is absorbed from the reservoir at $T_2$, $Q_1$ released to the reservoir at $T_1$ and $W = Q_2 - Q_1$ work done by the engine ( $W, Q_2, Q_1 > 0$ ) . Show that the efficiency $\frac{W}{Q_2}$ is the highest for reversible engines and that all reversible engines have the same efficiency.

**Q3. [Total Marks: 5+4+2]**

a.)An amount of ideal gas is kept under pressure and at a volume $V_1$. The wall of the container is made of good conductor of heat. The temperature of the gas is the same as the temperature of the atmosphere. The piston is suddenly pulled away and the volume of the gas is increased to $V_2$.

Calculate the total change in entropy of the universe; i.e. the gas and the environment.

b.) Suppose it is not ideal gas, but a gas with internal energy given be $u = cT - \frac{a}{v}$ (*c*, *a* are constants).

Describe how the entropy of the gas changes ( increases, decreases or remains the same) with time immediately following the pulling of the piston and subsequently as the gas stabilizes thermally with the environment.

In the same way describe the entropy change of the environment with time.

(detailed entropy change calculations are not required; but explanations necessary)

**Q4. [Total Marks: 3+3+6]**

a.) Derive the following equation: $dS = \frac{C_v}{T} dT + \left(\frac{\partial P}{\partial T}\right)_V dV$

b.) Show that $\left(\frac{\partial C_V}{\partial T}\right) = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$

c.) Using the above show that the entropy for a mole of gas with equation of state $P = \frac{RT}{v-b} - \frac{a}{v^2}$ is given by

$S = c_v \ln T + R \ln(v-b) + const$ provided $c_v$ is independent of T.

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Maxwell Relations that you may use or not use:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

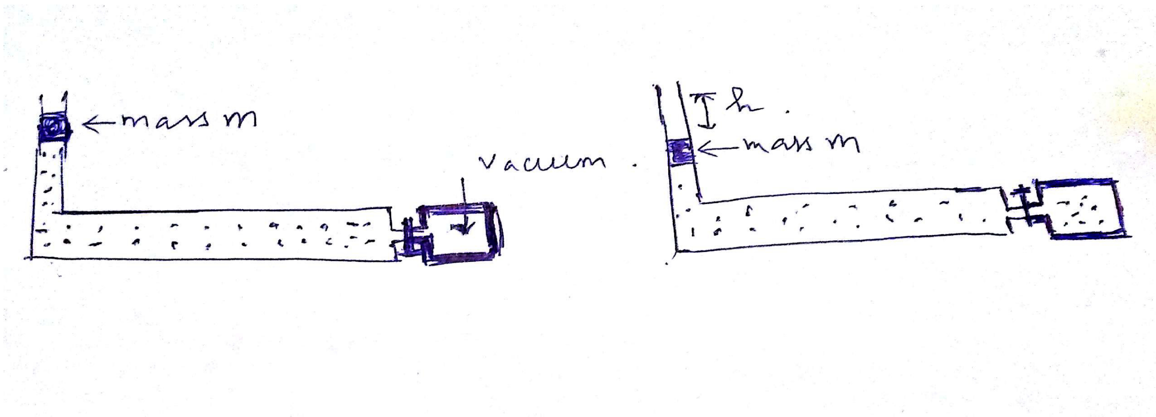


Fig 1

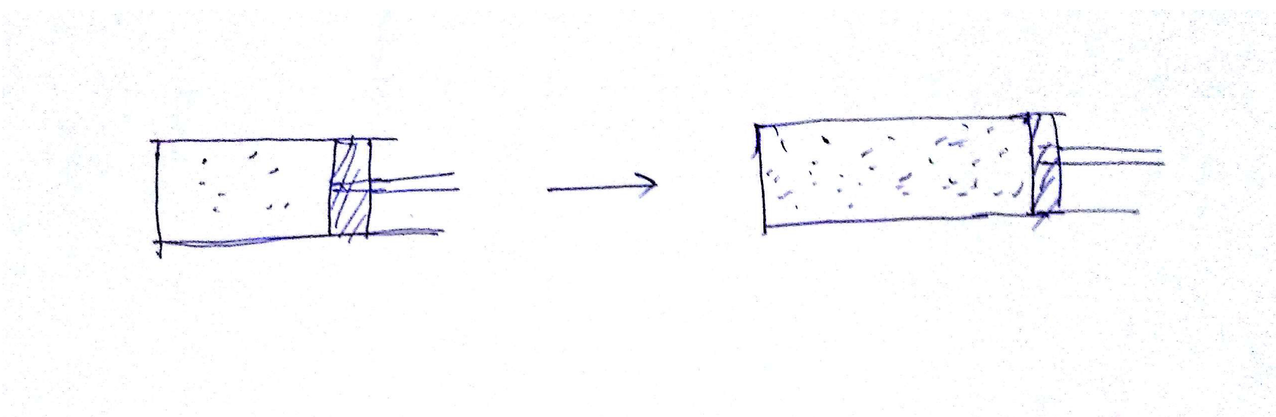

Fig 2